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ANALYTICAL STUDY OF THE PERFORMANCE OF A GUST ALLEVIATION SYSTEM WITH A VANE SENSOR

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16. Abstract

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The vane-controlled alleviation systems of the present study give better alleviation than did the accelerometer-controlled alleviation system of a previous study. However, the comparison is valid only for the assumptions made in both studies. Other factors should be considered in future work so that better judgment of both types of system can be made.

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SUMMARY

A 5670-kg airplane is assumed to be cruising with an airspeed of 109 m/sec at 3048-m altitude. It is also assumed that the airplane is flying in turbulent air characterized by a Von Kármán power spectral density function. Analysis has shown that a vane-controlled gust alleviation system (system I) in the airplane can reduce the normal acceleration at the airplane center of gravity by about 76 percent. The response to gusts of an angle-of-attack vane (the vane is located ahead of the wing) is fed simultaneously to the flap and elevator servos to provide alleviation. The signal to the servos is filtered so that required flap deflection rates are not too large. Also, another system (system II) uses a filter at the elevator servo to compensate for the time lead between vane response and elevator movement. This alleviation system has been effective in increasing the alleviation by 2.5 percent and 5 percent at stations 1 and 2 mean aerodynamic chords behind the airplane center of gravity, respectively, as compared to system I.

The vane-controlled alleviation systems of the present study give better alleviation than did the accelerometer-controlled alleviation system of a previous study. However, the comparison is valid only for the assumptions made in both studies. Other factors should be considered in future work so that better judgment of both types of system can be made.

INTRODUCTION

One method for alleviating the effect of air turbulence on an airplane is to deflect the wing flaps to offset the lift due to gusts and to deflect the elevator to offset the pitching motion due to gusts and flap deflections. Automatic operation of the flaps and elevator may be accomplished by forcing them to deflect proportionally to the response of a gust sensor. Gusts may be sensed by an accelerometer located at the airplane center of gravity or by an angle-of-attack vane located ahead of the wing. Although these are not the only gust sensors, automatic systems in which a vane or an accelerometer is used to operate the flaps and elevator have been shown, theoretically, to give good normal-acceleration alleviation (ref. 1).

The purpose of the present paper is to present a preliminary analysis of the performance of a vane-controlled gust alleviation system similar to the one studied in reference 1. The performance of the gust alleviation system is assessed by its ability to reduce the root-mean-square (rms) normal acceleration of an airplane flying in turbulent air. The atmospheric turbulence is assumed to be a random process with gust angles of attack characterized by a Von Kármán power spectral density function. It is also assumed that the random gust angle of attack is constant across the wing span and that the turbulence field is isotropic. The results of a similar study for an accelerometer-controlled gust alleviation system have been reported in reference 2.

The airplane, having a mass of 5670 kg and a wing loading of 145.3 kg/m^2, is assumed to be cruising at an airspeed of 109 m/sec and at an altitude of 3048 m. An angle-of-attack vane is mounted on a boom ahead of the wing. The response of the vane is first attenuated by a filter and then fed simultaneously to the flap and elevator servos. In addition, the pitch rate of the airplane is fed to the elevator servo. This gust alleviation system is compared with another one in which an additional filter is located between the vane and the elevator servo. The vane feedback gain is varied within a wide range of values to obtain the best performance. The results are compared with those of reference 2.

SYMBOLS

$A(s)$	servo transfer function for flaps and elevator
C_m	pitching-moment coefficient, $\dfrac{\text{Pitching moment}}{qS_w\bar{c}}$
C_Z	Z-force coefficient, $\dfrac{\text{Force in Z-direction}}{qS_w}$
$\bar{c}$	wing mean aerodynamic chord, m
$F(s), F_1(s)$	filter transfer functions in systems I and II (fig. 3)
g	free-fall acceleration, 9.80665 m/sec^2
$h(i\omega)$	frequency response function
$i = \sqrt{-1}$	
K_0	gain, rad/rad (rad/g from ref. 2)

K_2	gain
K_5	gain, $\frac{\text{rad}}{\text{rad/sec}}$
k_Y	radius of gyration about Y-axis, m
L	scale of turbulence, m
l_v	vane distance from airplane c.g., m
m	mass, kg
n	normal acceleration per g
q	dynamic pressure, N/m^2
R_0,R_1,R_2	alleviation, percent (eqs. (10))
S_w	wing area, m^2
s	complex variable, 1/sec
T,T_1	filter parameters in systems I and II (fig. 3), rad/sec
t	time, sec
V	airspeed, m/sec
w_g	vertical component of gust velocity at $\bar{c}/4$ (positive upward), m/sec
Z	complex matrix appearing in equation (A1)
$\left.\begin{matrix} z_{11},z_{12} \\ z_{21},z_{22} \end{matrix}\right\}$	complex numbers appearing in matrix of equation (A1)
α	angle of attack, rad
α_g	gust angle of attack, w_g/V, rad

δ_e	elevator deflection angle, rad
δ_f	flap deflection angle, rad
δ_v	vane deflection angle, rad
$\frac{d\epsilon}{d\alpha}$	change of downwash angle at tail with change of wing angle of attack
$\frac{d\epsilon}{d\delta_f}$	change of downwash angle at tail with change of flap deflection angle
θ	angle of pitch, rad
$\sigma_{n,0}$	rms normal acceleration per g at airplane c.g.
$\sigma_{n,1}$	rms normal acceleration per g at station $\bar{c}$ behind airplane c.g. along X-axis
$\sigma_{n,2}$	rms normal acceleration per g at station $2\bar{c}$ behind airplane c.g. along X-axis
σ_{w_g}	rms vertical component of gust velocity, m/sec
σ_{α}	rms angle of attack, rad
σ_{α_g}	rms gust angle of attack, rad
σ_{δ_e}	rms elevator deflection angle, rad
$\sigma_{\dot{\delta}_e}$	rms elevator deflection rate, rad/sec
σ_{δ_f}	rms flap deflection angle, rad
$\sigma_{\dot{\delta}_f}$	rms flap deflection rate, rad/sec
$\sigma_{\dot{\theta}}$	rms pitch rate, rad/sec
τ	transport time lag, $\frac{\text{Tail length}}{V}$, sec
τ_1	transport time lead, l_v/V, sec

$\Phi_{\alpha_g}(\omega)$ power spectral density function for gust angle of attack, $\frac{(\text{rad})^2}{\text{rad/sec}}$

ω circular frequency, rad/sec

$\left(C_{m_\alpha}\right)_o = \left(\frac{\partial C_m}{\partial \alpha}\right)_{\text{wing+fuselage}}$ $\left(C_{Z_\alpha}\right)_o = \left(\frac{\partial C_Z}{\partial \alpha}\right)_{\text{wing+fuselage}}$

$\left(C_{m_\alpha}\right)_t = \left(\frac{\partial C_m}{\partial \alpha}\right)_{\text{tail}}$ $\left(C_{Z_\alpha}\right)_t = \left(\frac{\partial C_Z}{\partial \alpha}\right)_{\text{tail}}$

$C_{m_{\delta_e}} = \frac{\partial C_m}{\partial \delta_e}$ $C_{Z_{\delta_e}} = \frac{\partial C_Z}{\partial \delta_e}$

$C_{m_{\delta_f}} = \frac{\partial C_m}{\partial \delta_f}$ $C_{Z_{\delta_f}} = \frac{\partial C_Z}{\partial \delta_f}$

Dots above symbols denote derivatives with respect to time.

ANALYSIS

Equations describing the longitudinal motion of an airplane flying at constant airspeed are used in the present investigation. A Von Kármán power spectral density function is used to represent the gust angle of attack. Application of random process theory gives the power spectral density function of the response of the airplane to the gust angle of attack. Automatic controls, actuated by feedback of a vane-angle signal and a rate-gyro signal, are modeled to alleviate the normal accelerations at the airplane center of gravity and at two other stations on the fuselage. The performance of the automatic controls is assessed by the percent reduction of root-mean-square (rms) normal acceleration obtained by operation of the controls.

Mathematical Model of the Airplane Motion

The equations of longitudinal motion for constant airspeed are used as in reference 1. Terms are included for forces and moments contributed separately by the wing plus fuselage, the horizontal tail, the elevator, and the flaps. This is done to account properly for the lag in downwash at the horizontal tail and for the effect of spatial distribution of vertical gusts. A disadvantage is incurred, however, because the resulting transport time lag leads to difficulties in the analysis of stability.

The frame of reference for the airplane motion is a system of body axes, as illustrated in figure 1. The differential equations are as follows:

$$m V\dot{\alpha}(t) = qS_W\left\{\left[\left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t\right]\alpha(t) - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\alpha}\alpha(t-\tau) + \left[\frac{mV}{qS_W} + \tau\left(C_{Z_\alpha}\right)_t\right]\dot{\theta}(t) + C_{Z_{\delta_e}}\delta_e(t)\right.$$

$$\left. + C_{Z_{\delta_f}}\delta_f(t) - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\delta_f}\delta_f(t-\tau) + \left(C_{Z_\alpha}\right)_o \alpha_g(t) + \left(C_{Z_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}\right)\alpha_g(t-\tau)\right\} \quad (1a)$$

$$m k_Y{}^2\ddot{\theta}(t) = qS_W\bar{c}\left\{\left[\left(C_{m_\alpha}\right)_o + \left(C_{m_\alpha}\right)_t\right]\alpha(t) - \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\alpha}\alpha(t-\tau) + \tau\left(C_{m_\alpha}\right)_t\dot{\theta}(t) + C_{m_{\delta_e}}\delta_e(t)\right.$$

$$\left. + C_{m_{\delta_f}}\delta_f(t) - \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\delta_f}\delta_f(t-\tau) + \left(C_{m_\alpha}\right)_o \alpha_g(t) + \left(C_{m_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}\right)\alpha_g(t-\tau)\right\}$$

(1b)

The Laplace transforms of equations (1) are

$$\left\{\frac{mVs}{qS_W} - \left[\left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\alpha}e^{-\tau s}\right]\right\}\alpha(s) - \left[\frac{mV}{qS_W} + \tau\left(C_{Z_\alpha}\right)_t\right]\dot{\theta}(s) - C_{Z_{\delta_e}}\delta_e(s)$$

$$- \left[C_{Z_{\delta_f}} - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\delta_f}e^{-\tau s}\right]\delta_f(s) = \left[\left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}\right)e^{-\tau s}\right]\alpha_g(s) \quad (2a)$$

$$\left[-\left(C_{m_\alpha}\right)_o - \left(C_{m_\alpha}\right)_t + \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\alpha}e^{-\tau s}\right]\alpha(s) + \left[\frac{m k_Y{}^2 s}{qS_W\bar{c}} - \tau\left(C_{m_\alpha}\right)_t\right]\dot{\theta}(s) - C_{m_{\delta_e}}\delta_e(s)$$

$$- \left[C_{m_{\delta_f}} - \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\delta_f}e^{-\tau s}\right]\delta_f(s) = \left[\left(C_{m_\alpha}\right)_o + \left(C_{m_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}\right)e^{-\tau s}\right]\alpha_g(s) \quad (2b)$$

The terms on the right-hand sides of equations (2) represent the gust disturbance where $\alpha_g = w_g/V$.

Gust Alleviation Systems

Equations (2) include terms with elevator and flap deflection angles δ_e and δ_f. The elevator and flaps are used as controls to alleviate the effect of turbulent air on the airplane normal acceleration and pitch rate. These controls are actuated automatically by a signal proportional to the angular displacement of a vane sensor. The vane is located ahead of the airplane wing as shown in figure 2. Sign convention is also shown in figure 2.

The equation for the vane response given as equation (23) in reference 1 becomes, in the notation of the present report,

$$\delta_V(t) = \tau_1\dot{\theta}(t) - \alpha(t) - \alpha_g(t + \tau_1) \tag{3}$$

The Laplace transform of equation (3) is

$$\delta_V(s) = \tau_1\dot{\theta}(s) - \alpha(s) - e^{\tau_1 s}\alpha_g(s) \tag{4}$$

Equation (3) indicates that the vane measures the angle of attack of the gust τ_1 seconds before the gust reaches the airplane center of gravity. This lead time $\left(\tau_1 = \frac{l_V}{V}\right)$ is the time required for the airplane to fly the distance between the vane and the airplane center of gravity.

The vane response is first attenuated by a first-order filter and then fed simultaneously to the flap and elevator servos. Additionally, the airplane pitch rate is fed to the elevator servo. A block diagram of this alleviation system (system I) is shown in figure 3(a). Equations for the flap and elevator responses are as follows:

$$\left.\begin{aligned}\delta_f(s) &= K_0F(s)A(s)\delta_V(s)\\ \delta_e(s) &= K_5A(s)\dot{\theta}(s) + K_2K_0F(s)A(s)\delta_V(s)\end{aligned}\right\} \tag{5}$$

If the filter $F(s)$ is adjusted to compensate for the lead time between the vane response and the flap response, it does not fully compensate for the longer lead time between the vane response and the elevator response. Consequently, an alternate alleviation system (system II) is considered which includes additional attenuation, by a first-order filter, of the vane response that is fed to the elevator servo. The location of this filter is shown in figure 3(b). The elevator-response equation for system II is

$$\delta_e(s) = K_5A(s)\dot{\theta}(s) + K_2F_1(s)K_0F(s)A(s)\delta_V(s)$$

Response to Turbulence

The vertical component of gust velocity w_g is assumed to be a random variable having a normal distribution with zero mean. The variance of the gust velocity is $\left(\sigma_{w_g}\right)^2$. The expression for the variance of the angle of attack caused by a vertical gust is

$$\left(\sigma_{\alpha_g}\right)^2 = \frac{\left(\sigma_{w_g}\right)^2}{V^2}$$

For homogeneous, isotropic turbulence having scale L, the Von Kármán power spectral density function of the gust angle of attack is given as a function of circular frequency ω (ref. 1). The function is

$$\Phi_{\alpha_g}(\omega) = \frac{L\left(\sigma_{\alpha_g}\right)^2}{\pi V^3} \frac{1 + \frac{8}{3}\left(1.339 \frac{L\omega}{V}\right)^2}{\left[1 + \left(1.339 \frac{L\omega}{V}\right)^2\right]^{11/6}} \tag{7}$$

The power spectral density of the output of the airplane is related to the power spectral density of gusts by the following result from random process theory:

$$\Phi_{output}(\omega) = \left|h(i\omega)\right|^2 \Phi_{\alpha_g}(\omega) \tag{8}$$

In equation (8), $\left|h(i\omega)\right|^2$ is the square of the absolute value of the frequency response function of an output variable. The frequency response functions are developed in the appendix.

The variance of the output is defined as the following integral:

$$\sigma_{output}^2 = \int_0^\infty \Phi_{output}(\omega)d\omega \tag{9}$$

Calculations

The airplane mass, dimensions, flight condition, and aerodynamic characteristics used in the present study are presented in table I. The flight condition is for cruise at 3048-m altitude. The scale of turbulence is assumed to be $L = 304.8$ m.

The filter parameter T (fig. 3) was adjusted until the rms flap deflection rate was reduced to a reasonable value. Then the vane position was adjusted to obtain the best alleviation. The resulting filter parameter T is 100 rad/sec, and the resulting vane location is $1.5\bar{c}$ ahead of the airplane center of gravity (2.48 m ahead of the wing leading edge). The results of reference 3 were used to estimate the flap- and elevator-servo characteristics. The filter parameter T_1 used in alleviation system II (fig. 3(b)) is the reciprocal of the time required for the airplane to fly the distance from the vane to the horizontal tail. Thus, T_1 is defined as

$$T_1 = \frac{V}{l_v + \text{Tail length}} = 10.2/\text{sec}$$

The values of the gains K_2 and K_5 are constant at 0.5 and at 0.5 $\frac{\text{rad}}{\text{rad/sec}}$, respectively. These gains were selected on the basis of preliminary calculations. Increasing the gain K_5 to values larger than 0.5 $\frac{\text{rad}}{\text{rad/sec}}$ does not give an appreciable increase in alleviation; and increasing the gain K_2 to values larger than 0.5 tends to cause system instability. These results agree with those of reference 2. The gain K_0 is varied from 1.2 to 2.5 rad/rad.

It is possible to do the calculations needed in the present analysis for a system that is unstable. A stability analysis was made to assure that all systems were stable. The transport time-lag and time-lead terms $e^{-\tau s}$ and $e^{\tau_1 s}$, respectively, make stability calculations extremely difficult. Therefore, stability was checked with $e^{-\tau s}$ approximated by $1 - \tau s$ and with $e^{\tau_1 s}$ approximated by $1 + \tau_1 s$. Both alleviation systems I and II are stable for the range of values of the gain K_0.

Equations (8) and (9) are used to obtain the rms normal acceleration per g at the center of gravity $\sigma_{n,0}$ and at $\bar{c}$ and $2\bar{c}$ behind the center of gravity $\sigma_{n,1}$ and $\sigma_{n,2}$. These results are used to obtain the percent reduction of normal acceleration, which is called alleviation, attributable to the alleviation system. The rms angle of attack σ_α, the rms pitch rate $\sigma_{\dot{\theta}}$, and the rms elevator and flap deflection angles σ_{δ_e} and σ_{δ_f}, and deflection rates $\sigma_{\dot{\delta}_e}$ and $\sigma_{\dot{\delta}_f}$ also are calculated. The rms vertical gust velocity used in the calculations is $\sigma_{w_g} = 0.3048$ m/sec. The rms of an output variable can be scaled for any multiple of 0.3048 m/sec.

The response of the basic airplane (i.e., when $K_0 = K_2 = K_5 = 0$) is used as a basis for assessment of the gust alleviation system. The basic-airplane response is

$$\sigma_{n,0} = 0.0287/\text{g}$$

$$\sigma_{n,1} = 0.0298/\text{g}$$

$$\sigma_{n,2} = 0.0313/\text{g}$$

$$\sigma_{\dot{\theta}} = 0.00257 \text{ rad/sec}$$

and

$$\sigma_\alpha = 0.00265 \text{ rad}$$

Percent alleviation, at the three fuselage stations, is calculated by using the following equations:

$$\left.\begin{aligned} R_0 &= 100\left(1 - \frac{\sigma_{n,0}}{0.0287}\right) \\ R_1 &= 100\left(1 - \frac{\sigma_{n,1}}{0.0298}\right) \\ R_2 &= 100\left(1 - \frac{\sigma_{n,2}}{0.0313}\right) \end{aligned}\right\} \tag{10}$$

DISCUSSION

The performance of the vane-controlled alleviation systems I and II is presented in figures 4 and 5. It should be emphasized that the performance is presented for only one flight condition. The 5670-kg airplane is assumed to be cruising with a constant airspeed of 109 m/sec at 3048-m altitude. Also, it should be emphasized that the servo and filter characteristics of the alleviation systems are not optimized, but they are realistic.

The performance of the vane-controlled alleviation system I is calculated as a function of the gain K_0. The percent alleviation at the three fuselage stations, the rms pitch rate, the rms elevator and flap deflection angles, and the rms elevator and flap deflection rates are presented in figure 4. The rms angle of attack, although not shown in figure 4, is essentially constant for the given range of values of K_0. The maximum alleviation is 75.5 percent at the center of gravity, and 69.5 percent and 59.0 percent at $\bar{c}$ and at $2\bar{c}$ behind the center of gravity, respectively. These values occur for a gain K_0 of about 2.25 rad/rad. The rms pitch rate $\sigma_{\dot{\theta}}$ varies almost linearly from 0.0015 to 0.0022 rad/sec as K_0 varies from 1.2 to 2.5 rad/rad. Values for $\sigma_{\dot{\theta}}$ in this range are less than the value of $\sigma_{\dot{\theta}} = 0.0026$ rad/sec for the basic airplane. The rms elevator and flap deflection angles and deflection rates are almost linear with K_0.

The performance of the vane-controlled alleviation system II is presented in figure 5. The maximum alleviation is 75.5 percent at the center of gravity and 72.0 percent and 65.7 percent at $\bar{c}$ and at $2\bar{c}$ behind the center of gravity, respectively. These values occur for a gain K_0 of 2.25 rad/rad. The rms pitch rate for the range of K_0 is less than the value for the basic airplane. The rms elevator deflection angle and deflection rate are almost linear with K_0.

For comparison, the performance of system I is also included in figure 5. At the center of gravity, the alleviation is practically the same for both systems for the range of K_0. However, at $\bar{c}$ and at $2\bar{c}$ behind the center of gravity, system II gives about

2.5 percent and 5.0 percent, respectively, more alleviation than does system I. This result suggests that the additional filter function $F_1(s)$ of system II is effective in reducing pitch accelerations. Also, from figure 5(d), system II controlled pitch rate slightly better than did system I. The rms elevator deflection angle and deflection rate required for alleviation are smaller for system II than for system I. The rms flap deflection angle and deflection rate are not included in figure 5 since they are the same for both systems. The desirability of including an additional filter in the vane-to-elevator feedback loop is obvious.

A comparison of the performance of the present vane-controlled alleviation systems with the similar accelerometer-controlled alleviation system of reference 2 can be made on only a narrow basis. The filter transfer functions of both types of system are designed to give good alleviation with a relatively low rms flap deflection rate. Therefore, the alleviations obtained for both types of system are logically compared for the same required flap deflection rate. This comparison is shown in figure 6(a). For most values of rms flap deflection rate, the vane-controlled alleviation system gives better alleviation at the center of gravity than does the accelerometer-controlled alleviation system. The maximum alleviation obtained by both types of system occurs for an rms flap deflection rate of about 0.0435 rad/sec. Figure 6(b) shows the corresponding alleviation at $\bar{c}$ and at $2\bar{c}$ behind the center of gravity. When $\sigma_{\dot{\delta}_f} = 0.0435$ rad/sec, the rms pitch rate, rms elevator deflection angle and deflection rate, and rms flap deflection angle are nearly the same for both types of system. Further research is needed to obtain a comparison of vane-controlled alleviation systems and accelerometer-controlled alleviation systems in which other important factors are considered. For instance, the effects of changes in airspeed and in gains on the alleviation and on the stability of the systems need to be explored as in reference 1. Furthermore, the possible use of other aerodynamic controls (i.e., spoilers, blown-flaps, etc.) should be considered in such a comparison. In any event, for the particular assumptions made, the present comparison is justified.

CONCLUDING REMARKS

A 5670-kg airplane is assumed to be cruising with an airspeed of 109 m/sec at 3048-m altitude. It is also assumed that the airplane is flying in turbulent air characterized by a Von Kármán power spectral density function. Analysis has shown that a vane-controlled gust alleviation system (system I) in the airplane can reduce the normal acceleration at the airplane center of gravity by about 76 percent. The response to gusts of an angle-of-attack vane (the vane is located ahead of the wing) is fed simultaneously to the flap and elevator servos to provide alleviation. The signal to the servos is filtered so that required flap deflection rates are not too large. Also, another system (system II) uses a filter at the elevator servo to compensate for the time lead between vane response and elevator movement. This alleviation system has been effective in increasing the alleviation by 2.5 percent and 5 percent at stations 1 and 2 mean aerodynamic chords behind the airplane center of gravity, respectively, as compared to system I.

The vane-controlled alleviation systems of the present study give better alleviation than did the accelerometer-controlled alleviation system of a previous study. However, the comparison is valid only for the assumptions made in both studies. Other factors should be considered in future work so that better judgment of both types of system can be made.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., November 8, 1973.

APPENDIX

FREQUENCY RESPONSE FUNCTIONS

Formulation of frequency response functions is necessary for the application of random process theory as used in the research of the present paper. These frequency response functions are obtained by combining equations (2) and (5), replacing s by $i\omega$, and solving the resulting equations for the transfer functions $\frac{\alpha}{\alpha_g}(i\omega)$ and $\frac{\dot{\theta}}{\alpha_g}(i\omega)$.

Equations (2) and (5) are written in complex matrix form as follows:

$$\left[Z(i\omega)\right]\begin{bmatrix}\frac{\alpha}{\alpha_g}(i\omega)\\ \\ \frac{\dot{\theta}}{\alpha_g}(i\omega)\end{bmatrix} = \begin{bmatrix}\frac{C_Z}{\alpha_g}(i\omega)\\ \\ \frac{C_m}{\alpha_g}(i\omega)\end{bmatrix} \tag{A1}$$

where the elements of the 2×2 complex matrix $Z(i\omega)$ are

$$z_{11}(i\omega) = \left(C_{Z_\alpha}\right)_o + \left(C_{Z_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}e^{-i\tau\omega}\right) - \frac{mV}{qS_W}i\omega$$

$$- \left[C_{Z_{\delta_f}} - \left(C_{Z_\alpha}\right)_t\frac{d\epsilon}{d\delta_f}e^{-i\omega\tau} + K_2C_{Z_{\delta_e}}\right]\frac{100K_0}{100 + i\omega}\,\frac{400\pi^2}{400\pi^2 - \omega^2 + 28\pi i\omega}$$

$$z_{12}(i\omega) = \left(C_{Z_\alpha}\right)_t\tau + \frac{mV}{qS_W} + \left\{\left[C_{Z_{\delta_f}} - \left(C_{Z_\alpha}\right)_t\frac{d\epsilon}{d\delta_f}e^{-i\tau\omega} + K_2C_{Z_{\delta_e}}\right]\frac{100}{100 + i\omega}\right.$$

$$\left. + K_5C_{Z_{\delta_e}}\right\}\frac{400\pi^2K_0\tau_1}{400\pi^2 - \omega^2 + 28\pi i\omega}$$

$$z_{21}(i\omega) = \left(C_{m_\alpha}\right)_o + \left(C_{m_\alpha}\right)_t\left(1 - \frac{d\epsilon}{d\alpha}e^{-i\tau\omega}\right) - \left[C_{m_{\delta_f}} - \left(C_{m_\alpha}\right)_t\frac{d\epsilon}{d\delta_f}e^{-i\tau\omega}\right.$$

$$\left. + K_2C_{m_{\delta_e}}\right]\frac{100K_0}{100 + i\omega}\,\frac{400\pi^2}{400\pi^2 - \omega^2 + 28\pi i\omega}$$

$$z_{22}(i\omega) = \left(C_{m_\alpha}\right)_t \tau - \frac{mk_Y^2}{qS_w\bar{c}} i\omega + \left\{\left[C_{m_{\delta_f}} - \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\delta_f} e^{-i\tau\omega} + K_2 C_{m_{\delta_e}}\right] \frac{100}{100 + i\omega}\right.$$

$$\left. + K_5 C_{m_{\delta_e}}\right\} \frac{400\pi^2 K_0 \tau_1}{400\pi^2 - \omega^2 + 28\pi i\omega}$$

The expressions for the terms on the right-hand side of equation (A1) are

$$\frac{C_Z}{\alpha_g}(i\omega) = -\left(C_{Z_\alpha}\right)_o - \left(C_{Z_\alpha}\right)_t \left(1 - \frac{d\epsilon}{d\alpha}\right) e^{-i\tau\omega} + \left[C_{Z_{\delta_f}} - \left(C_{Z_\alpha}\right)_t \frac{d\epsilon}{d\delta_f} e^{-i\tau\omega}\right.$$

$$\left. + K_2 C_{Z_{\delta_e}}\right] \frac{100 K_0}{100 + i\omega} \frac{400\pi^2}{400\pi^2 - \omega^2 + 28\pi i\omega} e^{i\tau_1\omega}$$

$$\frac{C_m}{\alpha_g}(i\omega) = -\left(C_{m_\alpha}\right)_o - \left(C_{m_\alpha}\right)_t \left(1 - \frac{d\epsilon}{d\alpha}\right) e^{-i\tau\omega} + \left[C_{m_{\delta_f}} - \left(C_{m_\alpha}\right)_t \frac{d\epsilon}{d\delta_f} e^{-i\tau\omega}\right.$$

$$\left. + K_2 C_{m_{\delta_e}}\right] \frac{100 K_0}{100 + i\omega} \frac{400\pi^2}{400\pi^2 - \omega^2 + 28\pi i\omega} e^{i\tau_1\omega}$$

The solution of equation (A1) gives the frequency response functions

$$\begin{bmatrix} \frac{\alpha}{\alpha_g}(i\omega) \\ \\ \frac{\dot{\theta}}{\alpha_g}(i\omega) \end{bmatrix} = \left[Z(i\omega)\right]^{-1} \begin{bmatrix} \frac{C_Z}{\alpha_g}(i\omega) \\ \\ \frac{C_m}{\alpha_g}(i\omega) \end{bmatrix} \tag{A2}$$

The frequency response functions for normal acceleration per g at three fuselage stations are

$$\frac{n_k}{\alpha_g}(i\omega) = -\frac{V}{g}\left[\frac{\dot{\theta}}{\alpha_g}(i\omega) - i\omega \frac{\alpha}{\alpha_g}(i\omega)\right] + \frac{k\bar{c}}{g} i\omega \frac{\dot{\theta}}{\alpha_g}(i\omega) \tag{A3}$$

where $k = 0,1,2$.

From equations (5), the frequency response functions for the elevator and flap deflection angles are obtained as follows:

$$\left.\begin{aligned}
\frac{\delta_f}{\alpha_g}(i\omega) &= \left[\tau_1 \frac{\dot{\theta}}{\alpha_g}(i\omega) - e^{i\tau_1\omega}\right]\frac{100K_0}{100 + i\omega}\,\frac{400\pi^2}{400\pi^2 - \omega^2 + 28\pi i\omega} \\
\frac{\delta_e}{\alpha_g}(i\omega) &= \left\langle\left(K_5 + K_2 \frac{100K_0\tau_1}{100 + i\omega}\right)\frac{\dot{\theta}}{\alpha_g}(i\omega) - \left[\frac{\alpha}{\alpha_g}(i\omega) + e^{i\tau_1\omega}\right]\frac{100K_2K_0}{100 + i\omega}\right\rangle\frac{400\pi^2}{400\pi^2 - \omega^2 + 28\pi i\omega}
\end{aligned}\right\} \tag{A4}$$

The frequency response functions for the elevator and flap deflection rates are simply

$$\left.\begin{aligned}
\frac{\dot{\delta}_f}{\alpha_g}(i\omega) &= i\omega\,\frac{\delta_f}{\alpha_g}(i\omega) \\
\frac{\delta_e}{\alpha_g}(i\omega) &= i\omega\,\frac{\delta_e}{\alpha_g}(i\omega)
\end{aligned}\right\} \tag{A5}$$

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1. Phillips, William H.; and Kraft, Christopher C., Jr.: Theoretical Study of Some Methods for Increasing the Smoothness of Flight Through Rough Air. NACA TN 2416, 1951.

2. Oehman, Waldo I.: Analytical Study of the Performance of a Gust Alleviation System for a STOL Airplane. NASA TN D-7201, 1973.

3. Barker, L. Keith: Effects of Spanwise Variation of Gust Velocity on Alleviation System Designed for Uniform Gust Velocity Across Span. NASA TN D-6346, 1971.

TABLE I.- AIRPLANE MASS, DIMENSIONS, FLIGHT CONDITION, AND AERODYNAMIC CHARACTERISTICS

Mass, m, kg	5669.905
Wing area, S_W, m^2	39.019
Mean aerodynamic chord, $\bar{c}$, m	1.981
Radius of gyration about Y-axis, k_Y, m	2.572
Tail length, m	7.742
True airspeed, V, m/sec	108.893
Altitude, m	3048
Dynamic pressure, q, N/m^2	5364.030
$\left(C_{Z_\alpha}\right)_o$, per rad	-5.3243
$\left(C_{Z_\alpha}\right)_t$, per rad	-0.682
$\left(C_{m_\alpha}\right)_o$, per rad	0.576
$\left(C_{m_\alpha}\right)_t$, per rad	-2.665
$C_{Z_{\delta_e}}$, per rad	-0.459
$C_{m_{\delta_e}}$, per rad	-1.813
$C_{Z_{\delta_f}}$, per rad	-2.292
$C_{m_{\delta_f}}$, per rad	0.430
$d\epsilon/d\alpha$	0.2884
$d\epsilon/d\delta_f$	0.133

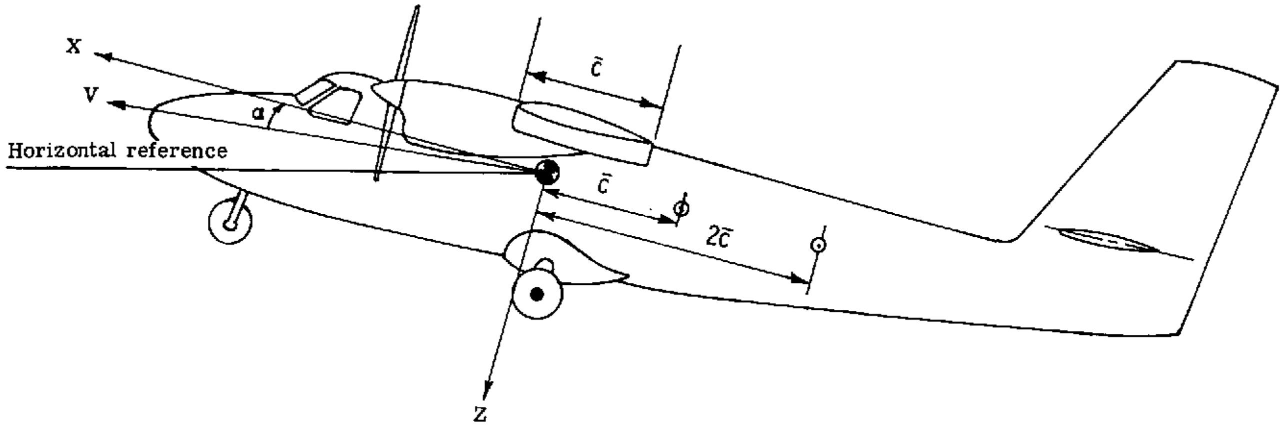


Figure 1.- Axis system.

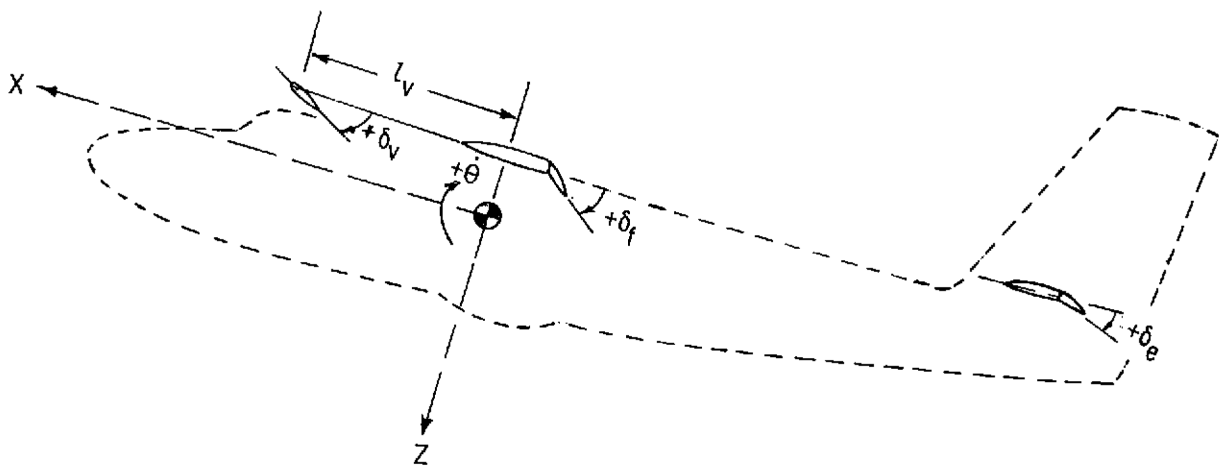
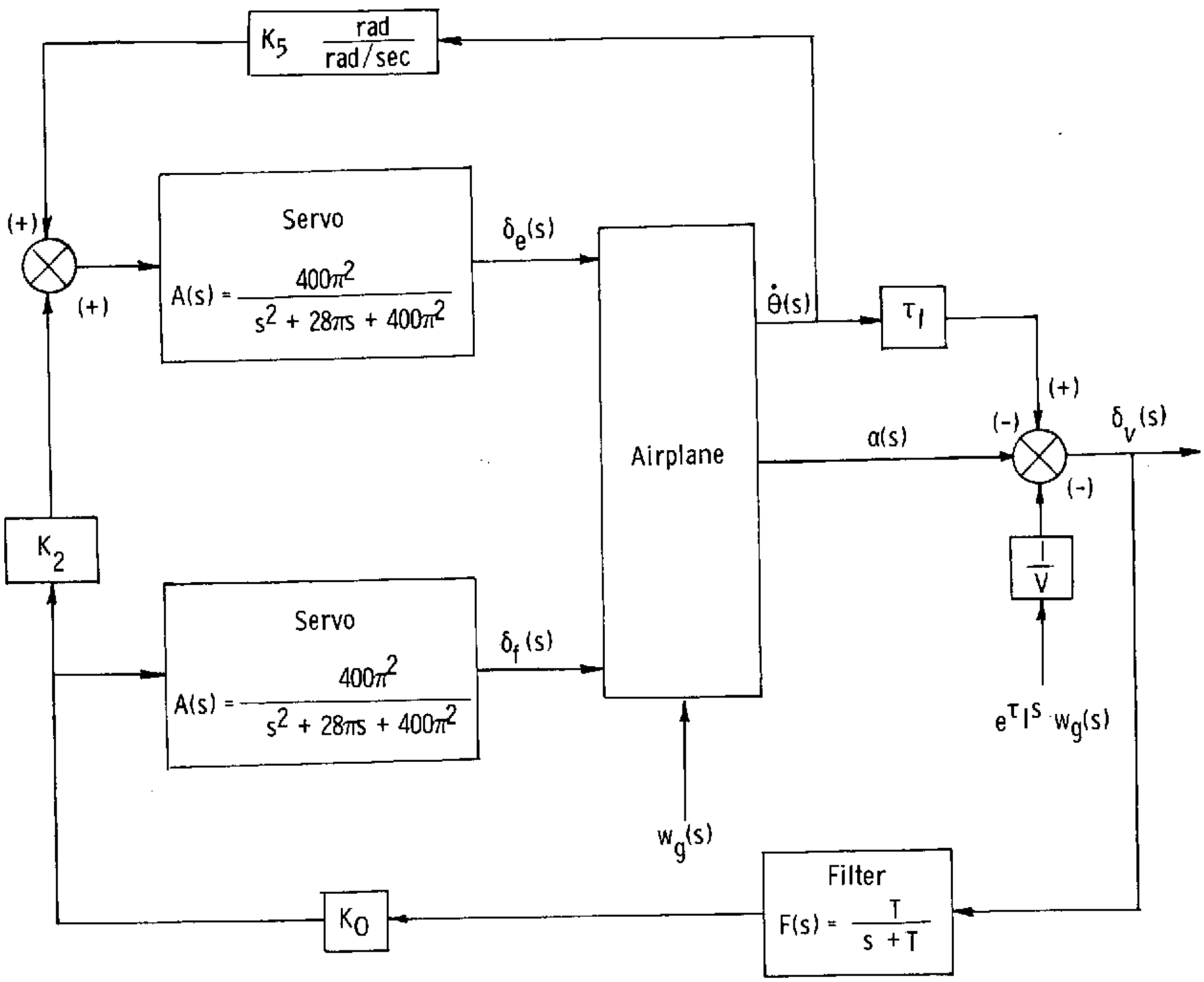
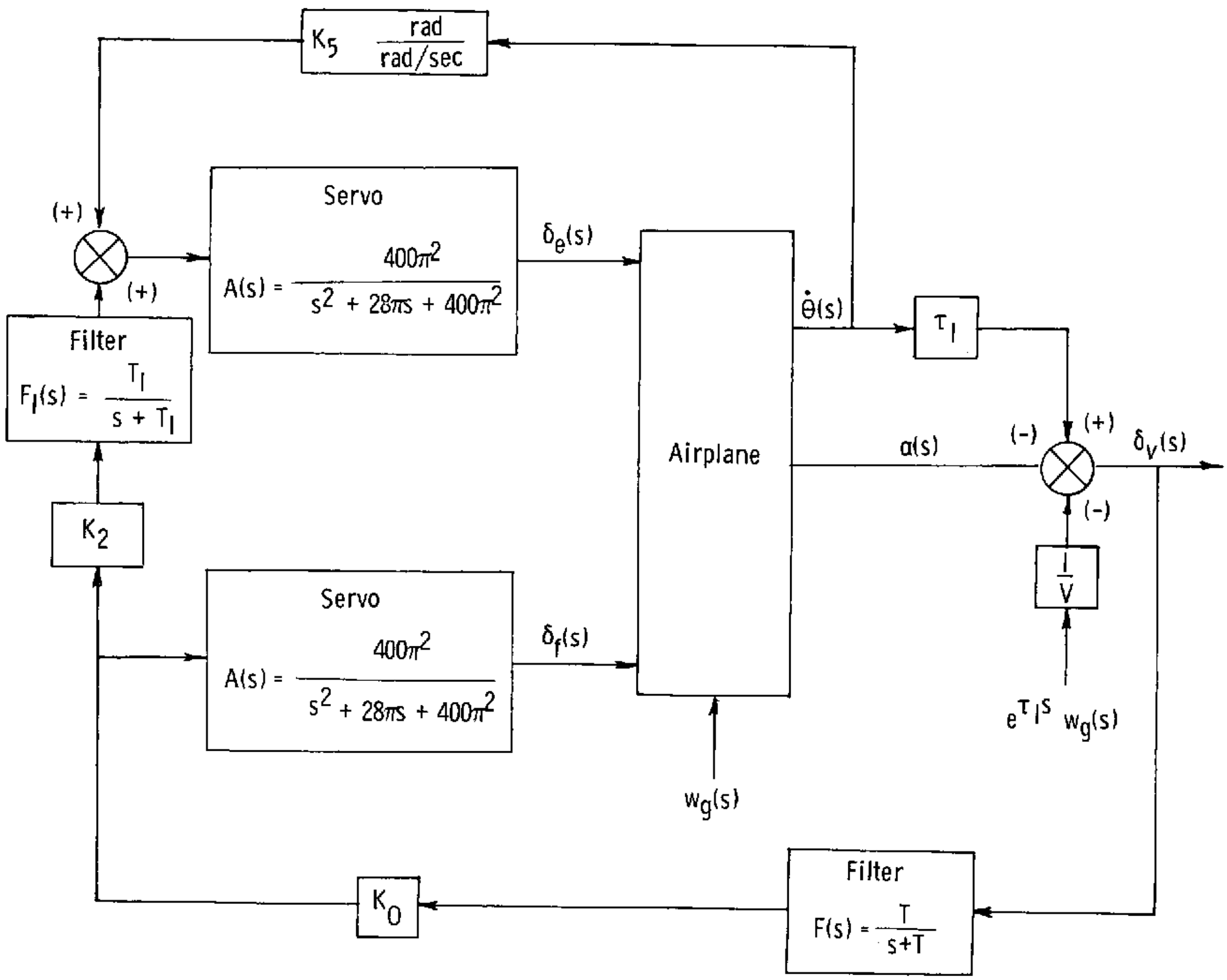


Figure 2.- Vane location and sign convention.



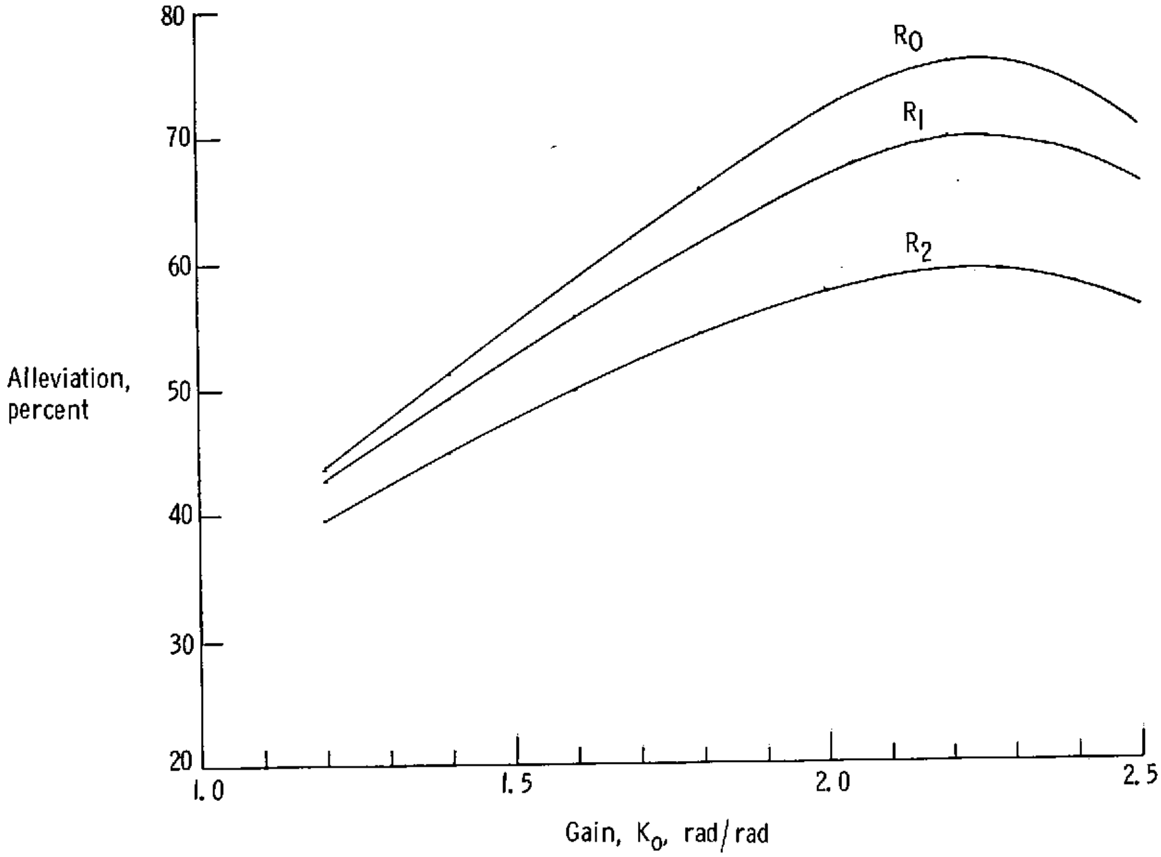
(a) System I.

Figure 3.- Block diagrams of gust alleviation systems.

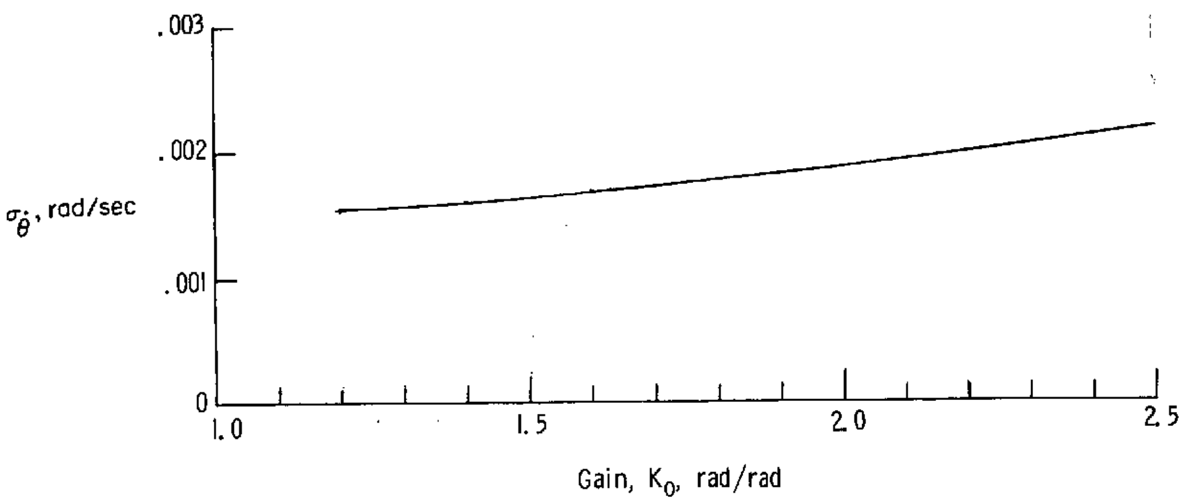


(b) System II.

Figure 3.- Concluded.

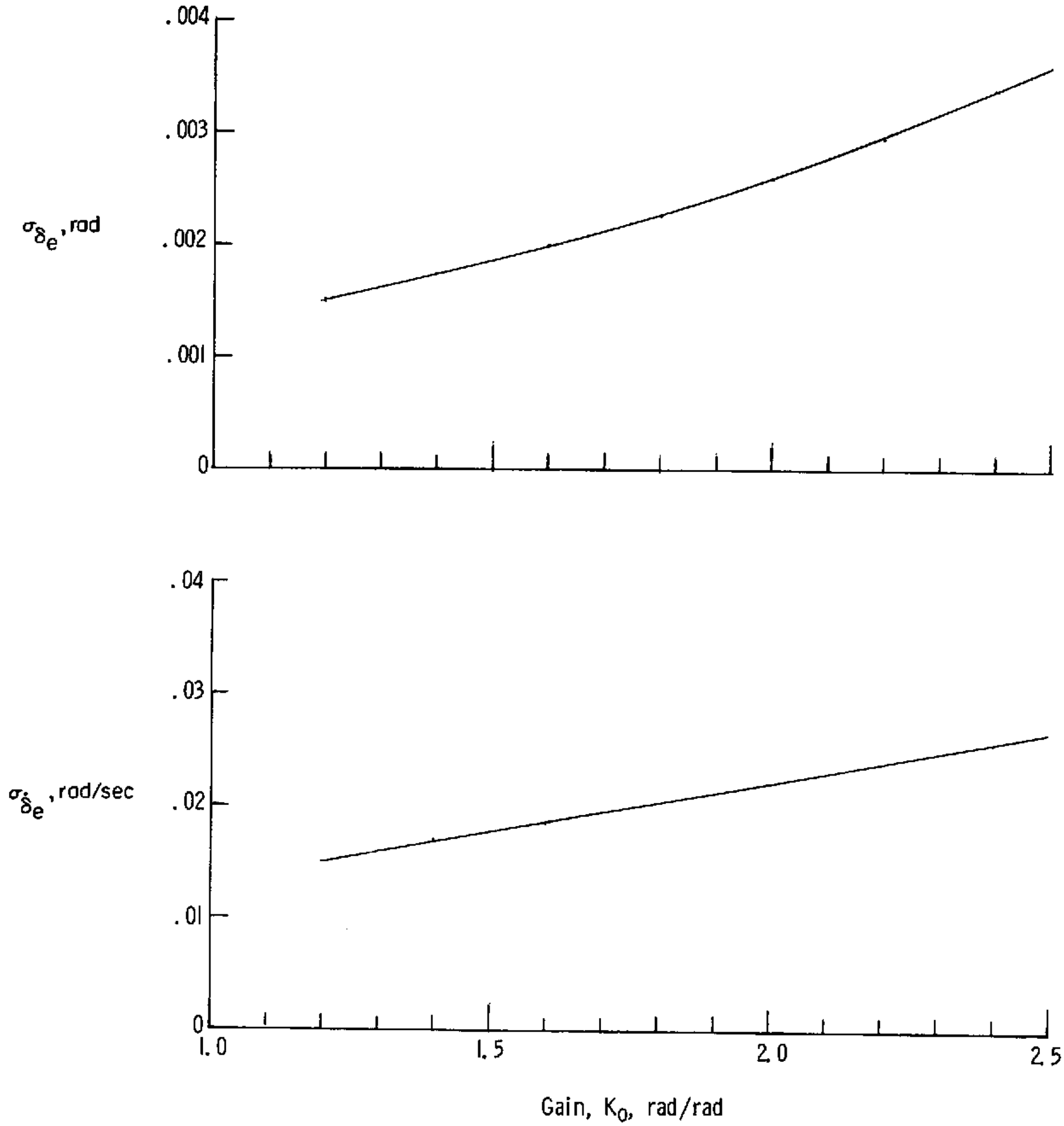


(a) Alleviation in normal acceleration.



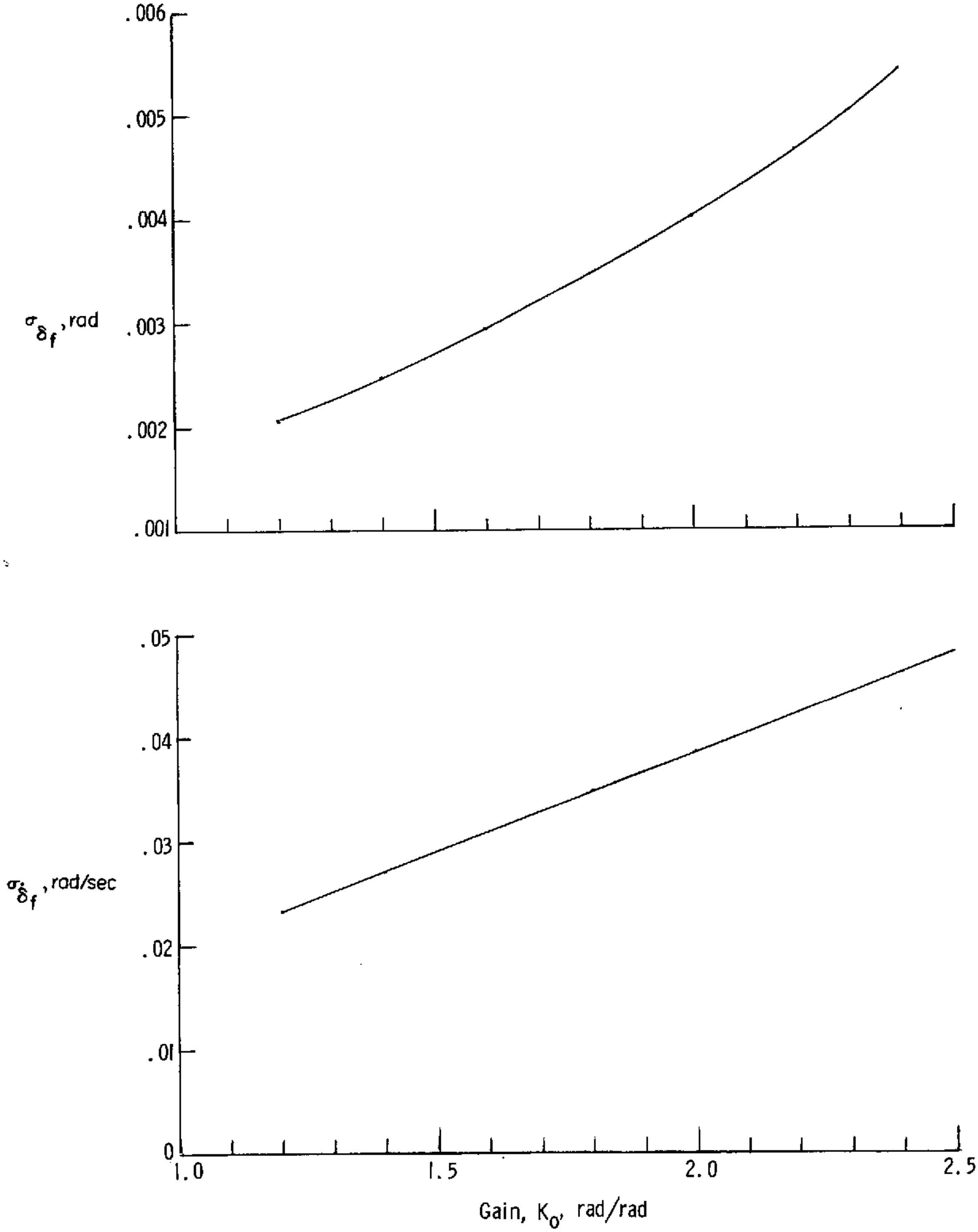
(b) rms pitch rate.

Figure 4.- Performance of gust alleviation system I. $\frac{l_v}{\bar{c}} = 1.5$; $K_2 = 0.5$; $K_5 = 0.5 \frac{\text{rad}}{\text{rad/sec}}$.



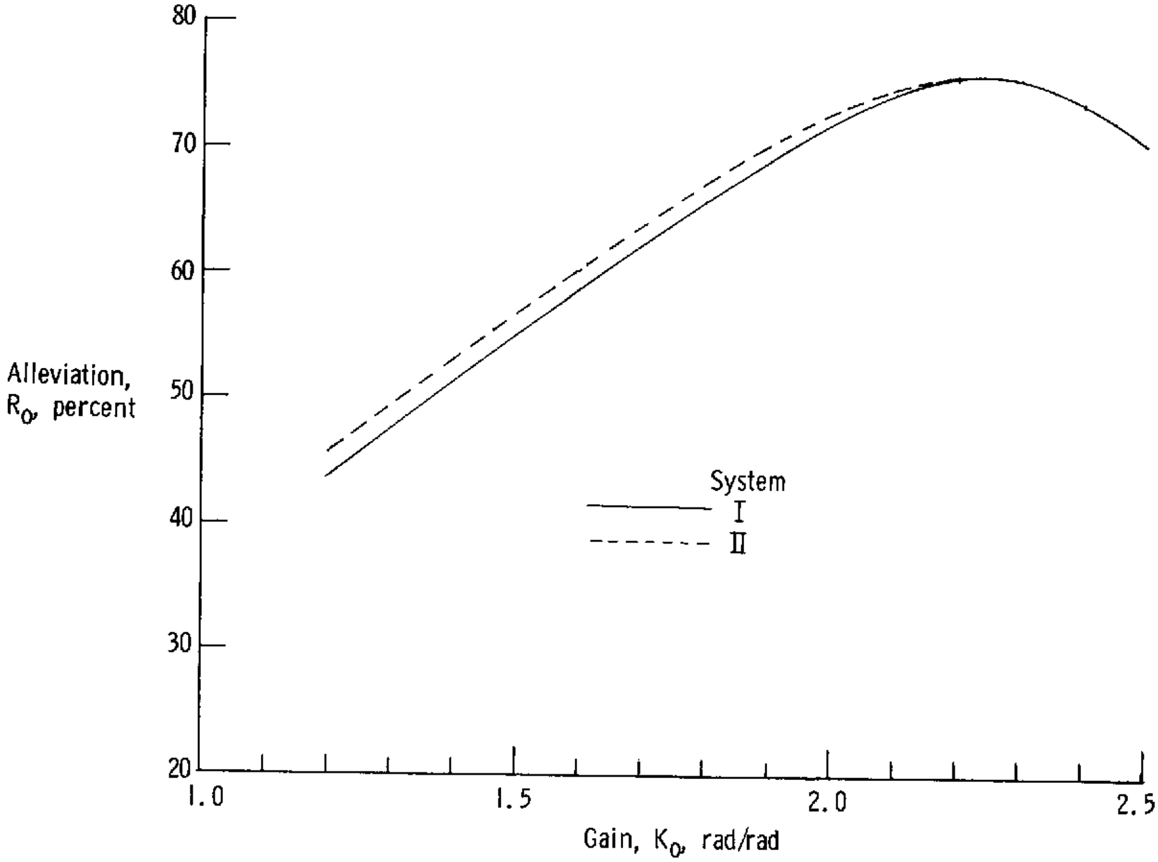
(c) rms elevator deflection angle and deflection rate.

Figure 4.- Continued.



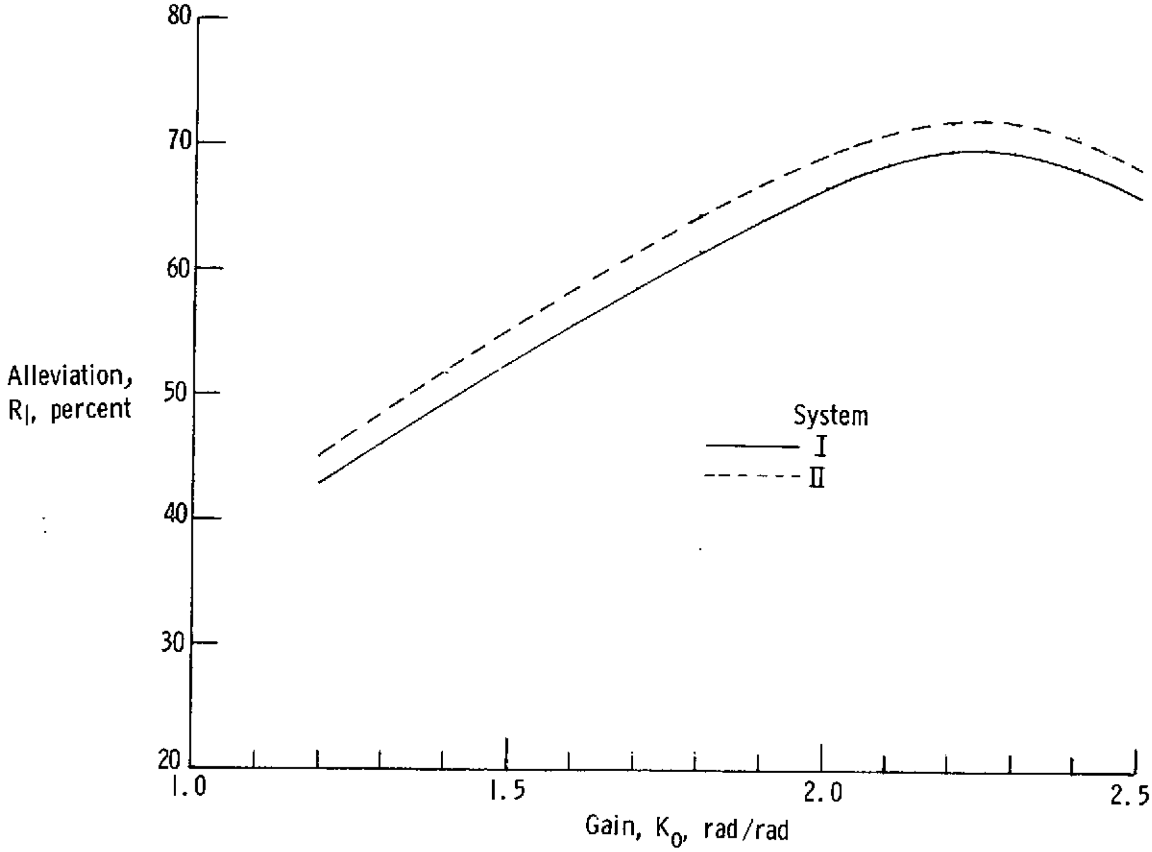
(d) rms flap deflection angle and deflection rate.

Figure 4.- Concluded.



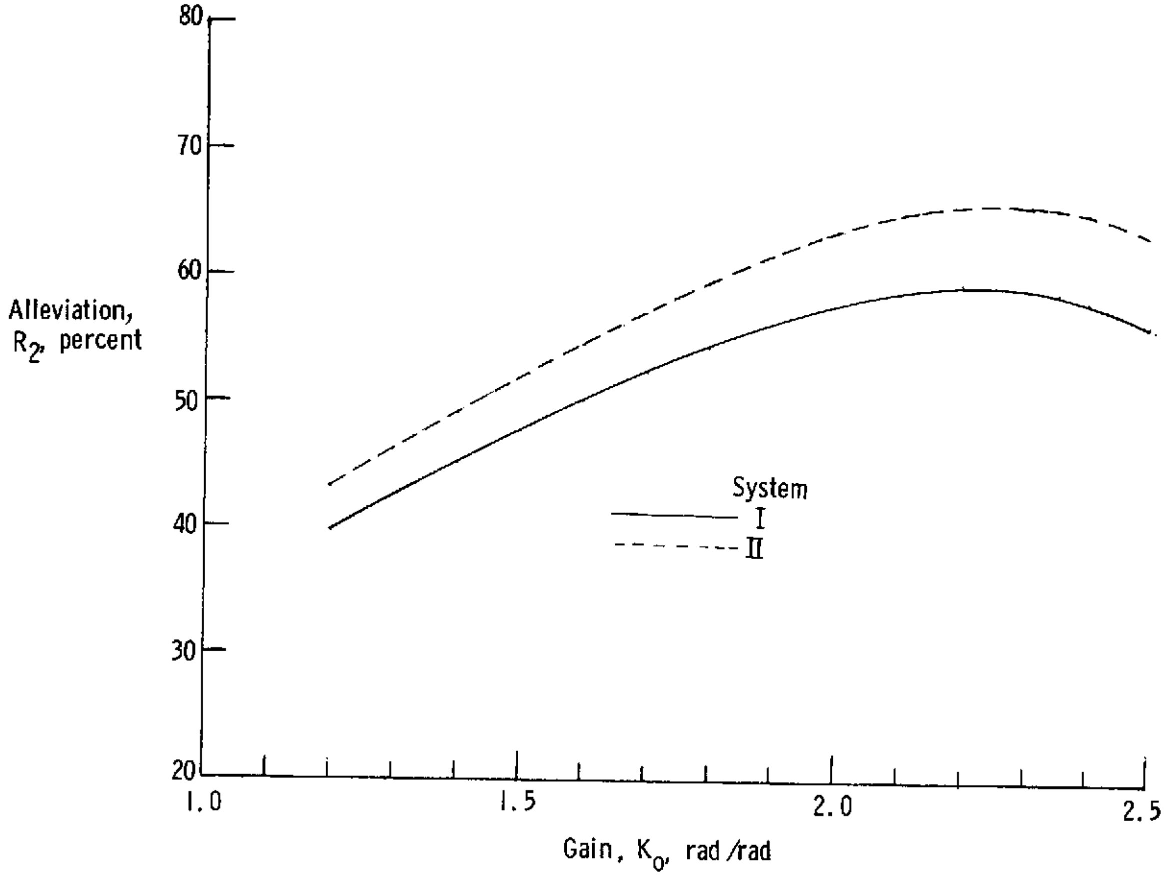
(a) Alleviation at airplane center of gravity.

Figure 5. - Performance of alleviation systems I and II.
$\frac{l_v}{\bar{c}} = 1.5$; $K_2 = 0.5$; $K_5 = 0.5 \frac{\text{rad}}{\text{rad/sec}}$.

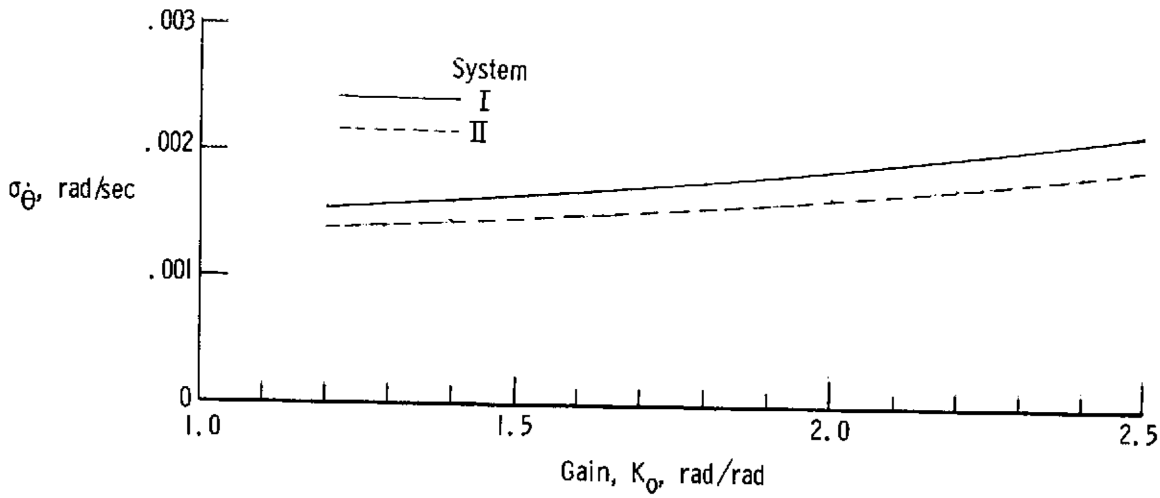


(b) Alleviation at $\bar{c}$ behind airplane center of gravity.

Figure 5.- Continued.

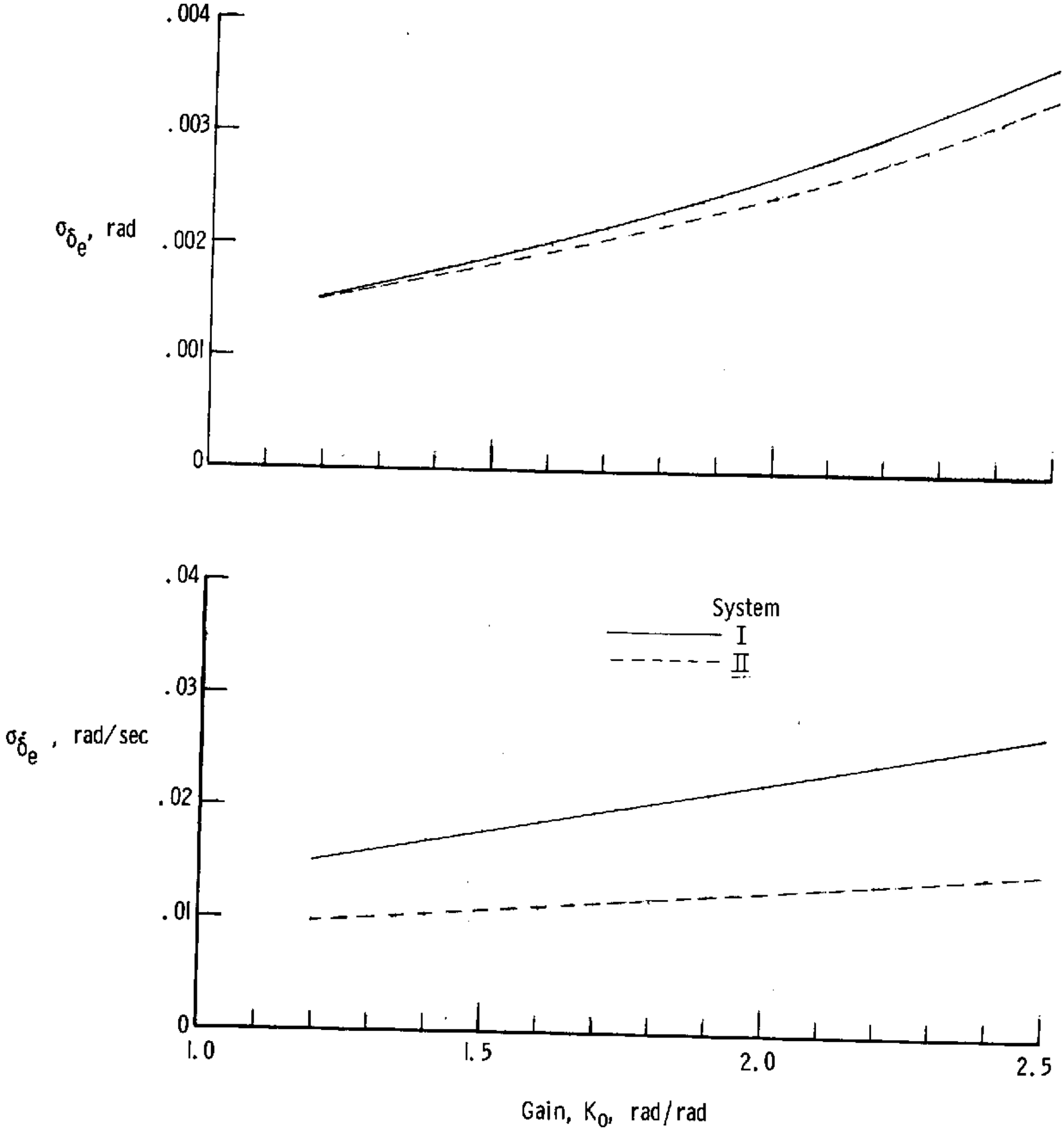


(c) Alleviation at $2\bar{c}$ behind airplane center of gravity.



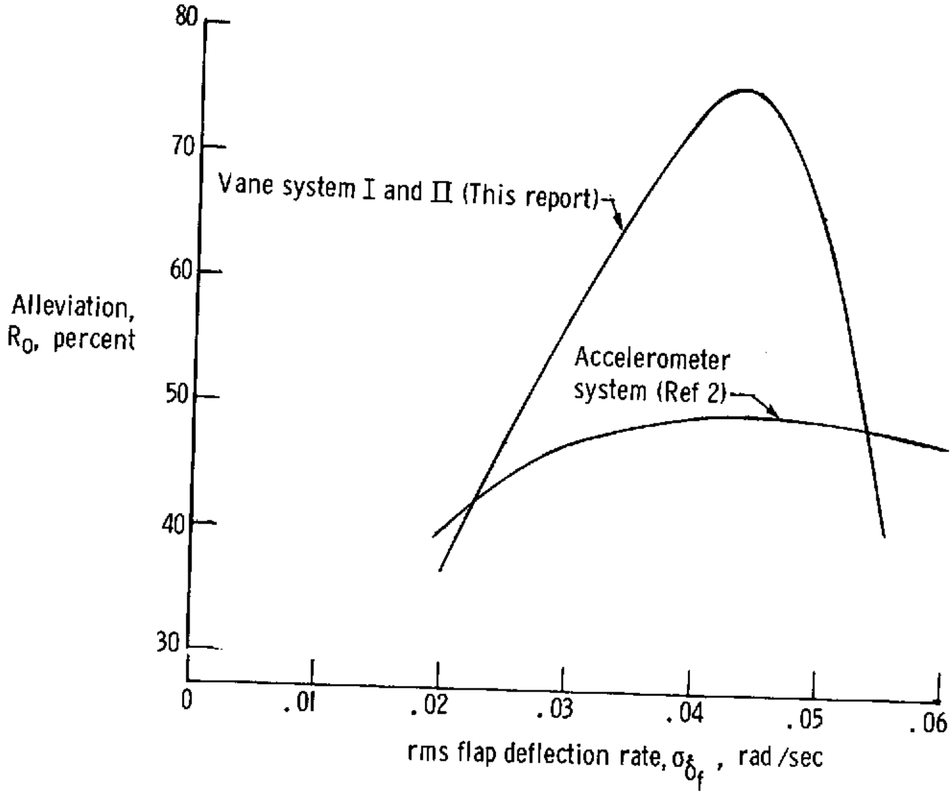
(d) rms pitch rate.

Figure 5.- Continued.



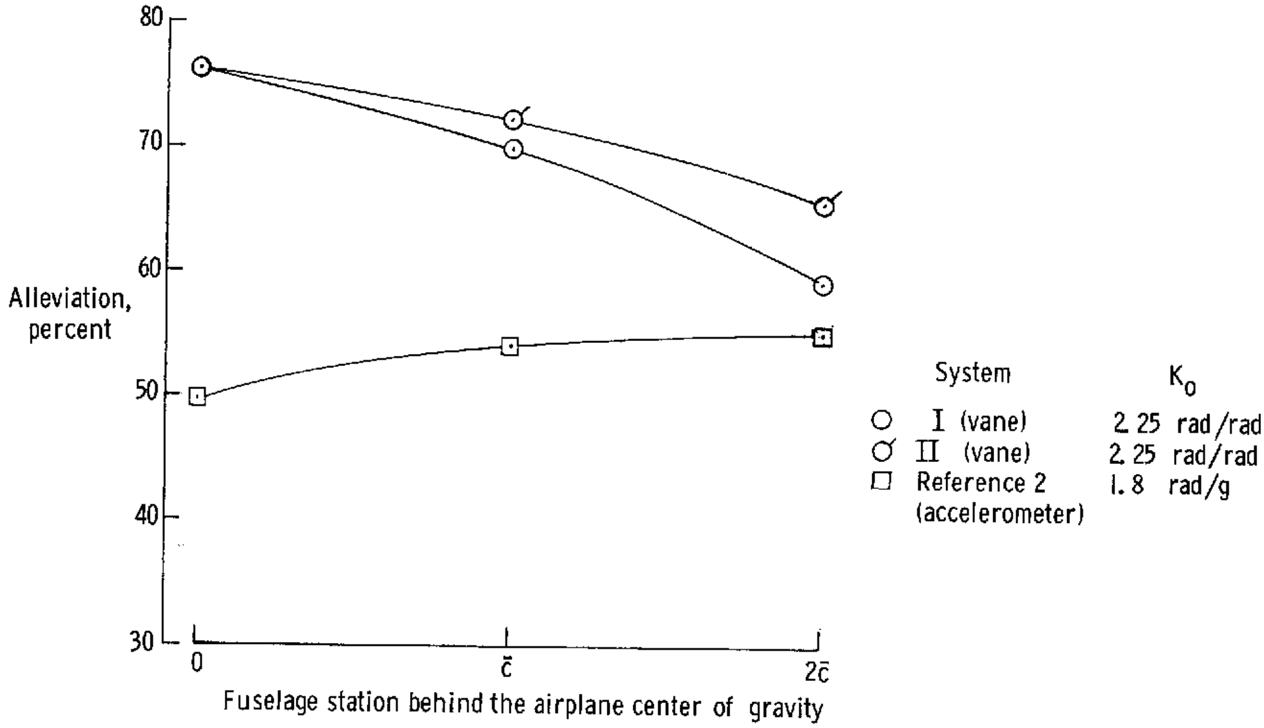
(e) rms elevator deflection angle and deflection rate.

Figure 5.- Concluded.



(a) Alleviation at airplane center of gravity.

Figure 6.- Vane-controlled alleviation systems compared with accelerometer-controlled alleviation system. $K_2 = 0.5$; $K_5 = 0.5 \frac{\text{rad}}{\text{rad/sec}}$.



(b) Alleviation behind airplane center of gravity. $\sigma_{\dot{\delta}_f} = 0.0435$ rad/sec.

Figure 6.- Concluded.